Fitting of the model angular spectrum of the free-free Galactic emission to the high-latitude WHAM data

1. The data

We use WHAM integral data (Madsen, Haffner, Reynolds, 2001) in the polar region to obtain the parameters of the model spectrum of the Galactic thermal emission (Chepurnov, 1999).

The original map was processed by the algorithm, equalizing background signal level in adjacent observation blocks (Chepurnov, 2004):

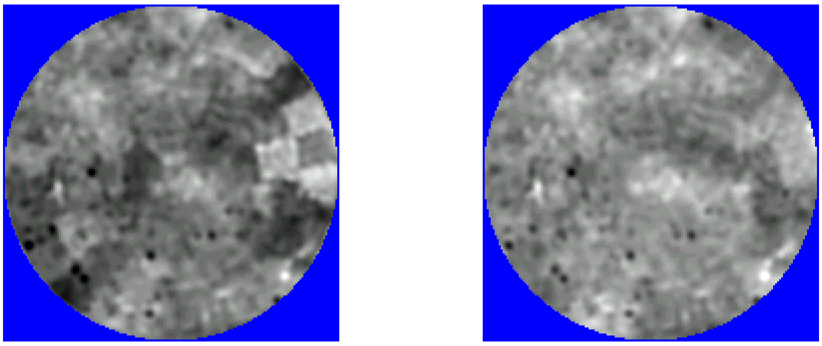

Figure 1. Used map region ($b = 90^\circ$, radius $= 25^\circ$). Uncleaned (left), cleaned (right).

2. Angular spectrum

Postulating Kolmogorov spectrum of electron density, the angular spectrum of the thermal Galactic emission can be expressed as follows (Chepurnov, 1999):

$$C_{ff}^2(\lambda, l) = 4.0 \cdot 10^{-10} \cdot \lambda^{4.32} \cdot \frac{\overline{n_e}^2 \cdot F_0^2 \cdot R^{8/3}}{l^{11/3} \cdot \left(1 + 0.5 \cdot \left(\frac{2\pi R}{l \cdot L}\right)^2\right)^{4/3}} +$$

$$+ 6.65 \cdot 10^{-10} \cdot \lambda^{4.32} \cdot \frac{F_0^4 \cdot R^{8/3}}{\left(\frac{2\pi}{L}\right)^{2/3}} \cdot \frac{1 + 3.21 \cdot e^{-1.43 \frac{l \cdot L}{2\pi R}}}{l^{11/3} \cdot \left(1 + 5.0 \cdot \left(\frac{2\pi R}{l \cdot L}\right)^2\right)^{4/3}}$$

where

$\overline{n_e}$, cm^{-3} is the mean electron density,

F_0^2, $m^{-20/3}$ is the electron density spatial spectrum amplitude,

R, pc – distance to the luminous volume border,

L, pc – injection scale,

λ, cm – wavelength,

the result is measured in K^2.

On the other hand, we can write for the average temperature T_0 at $\lambda = 1cm$ in the polar direction (Kaplan, Pikelner, 1979), accounting for Gaussian window function:

$$T_0 = 4.8 \cdot 10^{-6} \cdot \int_{l.o.s.} n_e^2 dr = 4.25 \cdot 10^{-6} \cdot \overline{n}_e^2 R_0$$

where R_0 is the gas-filled disk's half-thickness. According to the WHAM, T_0 has the following value:

$$T_0 = 2.56 \cdot 10^{-6}\, K$$

Consequently,

$$\overline{n}_e^2 = 2.35 \cdot 10^5 \cdot \frac{T_0}{R_0} \tag{2.1}$$

Accounting for this and introducing the following parameters:

$$p \equiv F_0^2 L^{5/3}, \tag{2.2}$$

$$q \equiv \frac{R_0}{L}, \tag{2.3}$$

$$\alpha \equiv \frac{R}{R_0} \tag{2.4}$$

we can rewrite the spectrum in the following form:

$$\begin{aligned} C_{ff}^2(\lambda, l) = 9.4 \cdot 10^{-5} \cdot \lambda^{4.32} \cdot p \cdot \frac{T_0 \cdot \alpha^{8/3} \cdot q^{5/3}}{l^{11/3} \cdot (1 + 0.5 \cdot (\frac{2\pi\alpha q}{l})^2)^{4/3}} + \\ + 1.95 \cdot 10^{-10} \cdot \lambda^{4.32} \cdot p^2 \cdot \frac{1 + 3.21 \cdot e^{-1.43 \frac{l}{2\pi\alpha q}}}{l^{11/3} \cdot (1 + 5.0 \cdot (\frac{2\pi\alpha q}{l})^2)^{4/3}} \end{aligned} \tag{2.5}$$

For α one can assume $\alpha = 1/\sin|b|$, where b is Galactic latitude.
Now we have only two free parameters in the spectrum, p and q, which we will estimate below.

3. The filter

We've used a filter that produces dispersion $D(\vec{\rho})$ of a signal $s(\vec{\rho}')$ in a circular region $\Omega_a(\vec{\rho})$ with radius a, centered at $\vec{\rho}$:

$$D(\vec{\rho}) = \frac{1}{\pi a^2} \int_{\Omega_a(\vec{\rho})} \left(s(\vec{\rho}') - \frac{1}{\pi a^2} \int_{\Omega_a(\vec{\rho})} s(\vec{\rho}'')\, d\vec{\rho}'' \right)^2 d\vec{\rho}' \tag{3.1}$$

Being applied to our free-free angular spectrum, it gives:

$$D = \frac{1}{(2\pi)^2}\int \Phi^2(a,\vec{\kappa})\, C_{ff}^2(1,\vec{\kappa})\, d\vec{\kappa} = p f_1(a,\alpha,q) + p^2 f_2(a,\alpha,q) \tag{3.2}$$

where

$$\Phi^2(a,\kappa) = \left(1-\left(\tfrac{2}{a\kappa}J_1(a\kappa)\right)^2\right)\cdot \varphi^2(r,\kappa)\,, \tag{3.3}$$

$\varphi^2(r,\kappa) = \left(\tfrac{2}{r\kappa}J_1(r\kappa)\right)^2$ – filter, responsible for averaging over a pixel,

r – pixel radius, in our case 0.5°.

$$f_1(a,\alpha,q) = 9.4\cdot 10^{-5}\cdot \frac{1}{2\pi}\int_2^\infty \frac{T_0\cdot \alpha^{8/3}\cdot q^{5/3}}{l^{11/3}\cdot(1+0.5\cdot(\frac{2\pi\alpha q}{l})^2)^{4/3}}\cdot \Phi^2(a,\kappa)\,\kappa\, d\kappa \tag{3.4}$$

$$f_2(a,\alpha,q) = 1.95\cdot 10^{-10}\cdot \frac{1}{2\pi}\int_2^\infty \frac{1+3.21\cdot e^{-1.43\frac{l}{2\pi\alpha q}}}{l^{11/3}\cdot(1+5.0\cdot(\frac{2\pi\alpha q}{l})^2)^{4/3}}\cdot \Phi^2(a,\kappa)\kappa\, d\kappa \tag{3.5}$$

4. Mutual probability distribution of *p* and *q*

We have formed two sets of circular regions in the map, with radii $a_1 = 2.273^\circ$ and $a_2 = 8.334^\circ$ and with amounts of 95 and 7 regions in each set respectively. We've calculated dispersions according to (3.1) along with their standard deviations for each set. That gave us $D_1 = 0.323 \pm 0.026\,\mu K^2$ and $D_2 = 0.433 \pm 0.038\,\mu K^2$ at wavelength $\lambda = 1cm$.

This allowed us to estimate mutual probability distribution of D_i, roughly assuming that both variables have normal distribution and are independent:

$$P_D(D_1,D_2) = \frac{1}{2\pi\sigma_{D_1}\sigma_{D_2}} e^{-\frac{(D_1-\overline{D_1})^2}{2\sigma_{D_1}^2}} e^{-\frac{(D_2-\overline{D_2})^2}{2\sigma_{D_2}^2}} \tag{4.1}$$

Having solved (3.2) with respect to p, we obtain:

$$p = F(a_i,\alpha_i,q,D_i)\,, \qquad i = 1, 2 \tag{4.2}$$

where

$$F(a_i,\alpha_i,q,D_i) \equiv \frac{\sqrt{f_1^2(a_i,\alpha_i,q)+4f_2(a_i,\alpha_i,q)\cdot D_i} - f_1(a_i,\alpha_i,q)}{2f_2(a_i,\alpha_i,q)} \tag{4.3}$$

Equations (4.2) define dependence of (p,q) on (D_1,D_2). We will use them to derive probability distribution $P(p,q)$ as follows.

First of all, we write the elementary probability in the $\{(D_1,D_2)\}$ space:

$$dP = P_D(D_1, D_2)\, dD_1\, dD_2$$

It can be shown, that accounting for (4.2), the area element $dD_1 dD_2$ can be mapped to the $\{(p,q)\}$ space with the following expression:

$$dS = \frac{\frac{\partial}{\partial D_1} F(a_1, \alpha_1, q, D_1) \cdot \frac{\partial}{\partial D_2} F(a_2, \alpha_2, q, D_2)}{\left| \frac{\partial}{\partial q} F(a_1, \alpha_1, q, D_1) - \frac{\partial}{\partial q} F(a_2, \alpha_2, q, D_2) \right|} \cdot dD_1 dD_2$$

Finally we have:

$$P(p,q) = \frac{dP}{dS} = \frac{\left| \frac{\partial}{\partial q} F(a_1, \alpha_1, q, D_1) - \frac{\partial}{\partial q} F(a_2, \alpha_2, q, D_2) \right|}{\frac{\partial}{\partial D_1} F(a_1, \alpha_1, q, D_1) \cdot \frac{\partial}{\partial D_2} F(a_2, \alpha_2, q, D_2)} \cdot P_D(D_1, D_2) \qquad (4.4)$$

with substitution of

$$D_i = p f_1(a_i, \alpha_i, q) + p^2 f_2(a_i, \alpha_i, q) \qquad (4.5)$$

5. Obtaining spectrum parameters

Integrating $P(p,q)$ over p or q we have probability distributions $P(q)$ and $P(p)$:

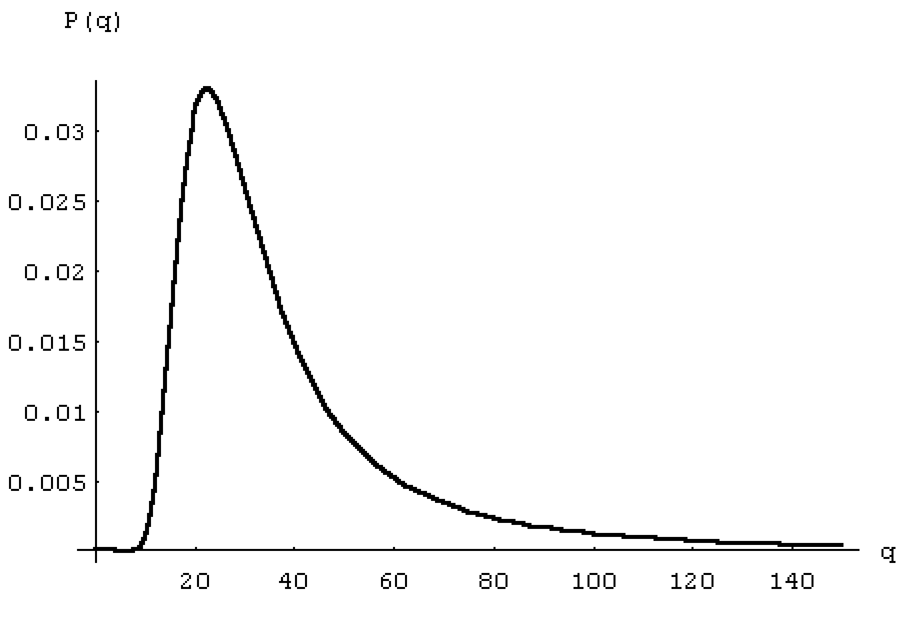

Figure 2. Probability distribution of the spectrum parameter q

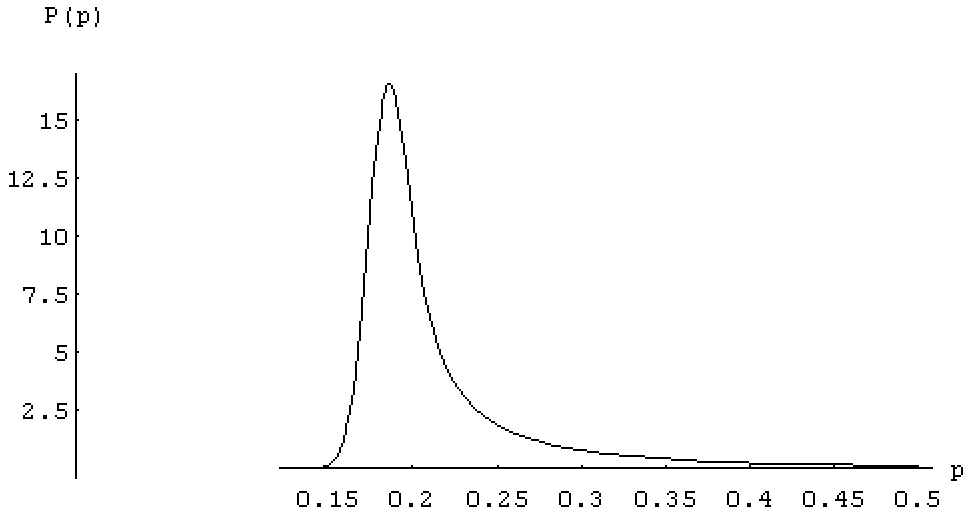

Figure 3. Probability distribution of the spectrum parameter p

Initially both curves have had the area of 0.81, which means that the $\{(p,q)\}$ space, mapped through the transformation (4.2), does not cover all areas in $\{(D_1,D_2)\}$, where $P_D(D_1,D_2)$ is significantly non-zero. This, in turn, means that the model spectrum is inconsistent with the observational data with probability 0.19.

Assuming that this is not the case, we've re-normalized $P(q)$ and $P(p)$. The resulting functions are shown in Figure 2 and Figure 3.

The behavior of $P(q)$ is not so good: $P(q) \propto q^{-2.8}$ for $q \geq 70$. This means that the first moment converges slowly, and the second moment does not exist. So for estimating of the q parameter we have to choose the median value, and for confidence interval – quantiles. Finally we have:

$$q = 31.3^{+27}_{-11} \tag{5.1}$$

for confidence probability 0.68.

Contrary to $P(q)$, $P(p)$ doesn't show any peculiarities and we can directly calculate its moments:

$$p = 0.219 \pm 0.068 \tag{5.2}$$

6. Summary

Using the WHAM data for $b > 65^\circ$ we have obtained free parameters (5.1) and (5.2) of the Galactic thermal emission angular spectrum (2.5). It appeared, that the spectrum is consistent with the observational data with the probability 0.81. The spectrum parameters are bound with the physical characteristics of the interstellar medium through equations (2.2), (2.3) and (2.1).

In particular, setting $R_0 = 1000\,pc$, we have:

$$L = 32^{+18}_{-14}\ pc$$
$$F_0^2 = 6.8^{+11.0}_{-3.6} \cdot 10^{-4}\ m^{-20/3}$$

for injection scale and amplitude of electron density spectrum.

The latter value is quite similar to $F_0^2 = 3.16 \cdot 10^{-4}\ m^{-20/3}$, found from pulsar scintillation studies by Cordes et al (1991).

The discussed angular spectrum exhibits asymptotic behavior, predicted independently by Lazarian (Jungyeon, Lazarian, 2002) and explains the observed angular spectral index behavior in the case of quadratic emissivity law, mentioned in that paper.

References

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6. Cordes J.M. et al, Nature, **354**, 121 1991.