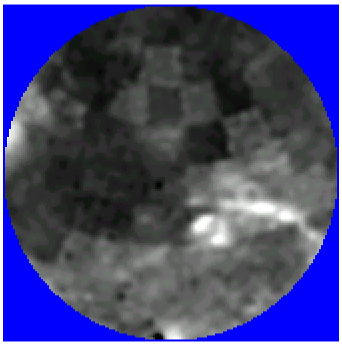
Removing systematic error from WHAM integral map

It's known, that WHAM integral survey has a systematic error due to variation of background signal level in different observation blocks (Madsen, Haffner, Reynolds 2001). It is clearly seen in low-signal areas:



$l = 258^\circ$, $b = 58^\circ$, brightness range 2.5R

We'll try to recover for it with the assumption that the error is additive.

Determining block corrections

Let's denote:

c_{ij} – initial signal level difference between i-th and j-th block, if they have a common border; $c_{ji} = -c_{ij}$

d_i – correction displacement to be found for i-th block

w_{ij} – weight for adjusting of an involvement of ij-border; $w_{ji} = w_{ij}$

$\Omega \equiv \{(i,j) \mid \exists c_{ij}\}$

$\Omega_i \equiv \{j \mid \exists c_{ij}\}$

A function to minimize is as follows:

$$L = \sum_{\Omega} \frac{1}{w_{ij}^2}(c_{ij} + d_i - d_j)^2$$

$$\frac{\partial L}{\partial d_n} = \sum_{\Omega_n} \frac{4}{w_{ij}^2}(c_{ij} + d_i - d_j) = 0$$

So we have a set of linear equations for d_i:

$$d_n \sum_{\Omega_n} \frac{1}{w_{nj}^2} - \sum_{\Omega_n} \frac{1}{w_{nj}^2} d_j = -\sum_{\Omega_n} \frac{c_{nj}}{w_{nj}^2} \qquad (1)$$

which is obviously linear-dependent.

One opportunity to avoid this is fixing at least one block by setting the respective displacement to zero. Below we'll see how these blocks are chosen as well as the other details of the algorithm.

Determining signal differences

Let us consider a function $y(\vec{r})$ defined on a set $A \cup B$, while its values on sets A and B differ by a constant. With exception of this difference, the function is considered to be continuous and smooth enough.

We are going to estimate this difference when given the function values y_i on some discrete grid $\{\vec{r}_i\}$.

Let us approximate $y(\vec{r})$ using a polynomial functional basis $\{f_n(\vec{r})\}$, $n = 1...N$, with account for the fact, that the constant term is different in A and B. Let's denote the values of this term c_A and c_B respectively, and c_n – other expansion coefficients, equal in both sets.

Having applied the method of least squares, we obtain the following system of $N+2$ equations for c_n, c_A and c_B:

$$\left(\begin{array}{ccc|c} \sum\limits_{A\cup B} f_n(\vec{r}_i) f_m(\vec{r}_i) & \sum\limits_{A} f_m(\vec{r}_i) & \sum\limits_{B} f_m(\vec{r}_i) & \sum\limits_{A\cup B} y_i f_m(\vec{r}_i) \\ \sum\limits_{A} f_n(\vec{r}_i) & M_A & 0 & \sum\limits_{A} y_i \\ \sum\limits_{B} f_n(\vec{r}_i) & 0 & M_B & \sum\limits_{B} y_i \end{array}\right)$$

where M_A and M_B are numbers of points in A and B.

Having solved it we take $c_A - c_B$ as the required difference.

Realization

Let an object "spot" represent one observational point, so it has a magnitude, coordinates and a number of a block it belongs to. Besides this let it maintain a list of other spots falling into its vicinity with given radius.

Having this information, a spot must be able to answer if it belongs to a border, and, if yes, to provide relevant block numbers and respective signal difference. After averaging of these differences over a border we have values of c_{ij}, which together with weights w_{ij} are used to form the system of equations (1) for block corrections.

In the present realization we've taken the weight inversely proportional to a mean signal over the two blocks to correct the algorithm's tendency to solve problems of high-magnitude blocks at a cost of distortion of low-signal areas.

However, sometimes it happens, that the calculation of a signal difference gives a false result because of a compact bright feature near the border. To avoid this, we've set a threshold for signal variation within a block. When exceeding it, the block along with all its neighbors is declared fixed. As a by-product, doing this, we make the set of equations linear independent.

Actually, that isn't an optimal decision. As one may see, in this case the blocks excluded from tuning occasionally work as reference blocks, which amplitudes are fixed and which actually determine the background level. This mixing of roles results in the fact, that the background signal level becomes less reliable than it could be, especially its absolute value.

It would be much better to examine the quality of difference calculation, and to declare the border between involved blocks fixed, if it were poor. Then we could glue the blocks and recalculate. It wouldn't make the system linear independent, so this way we separate the problem of unreliable difference calculation from the problem of setting up the background level.

For the latter we have at least two options.

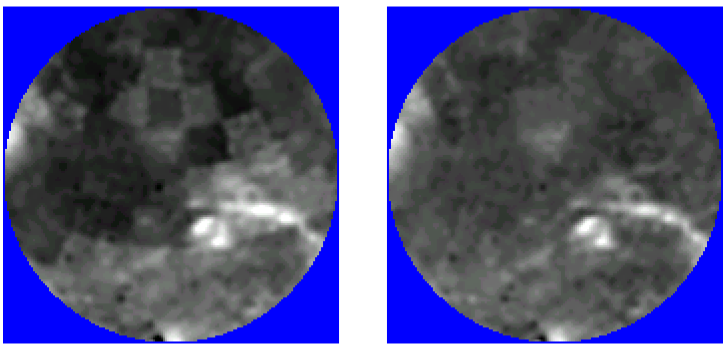
We could demand that weighted mean signal over the sky should remain constant and replace one of the equations with this condition. But it doesn't guarantee from other background distortions, which leave this level constant. However, it is one of the opportunities to try, weighting gives some room to adjustment.

Another option is to select the reference blocks by hand, accounting for their reliability ant trying to distribute them uniformly over the map. This variant seems to be more productive.

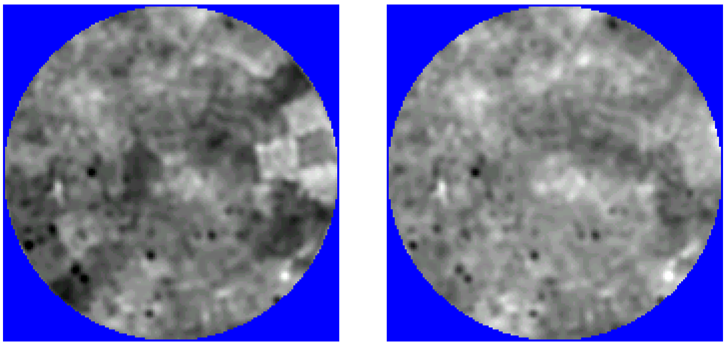
Intermediate result

While the algorithm is still to be refined, the current version gave a result, which can be used for applications where the absolute level isn't important.

Some examples:



$l = 258^\circ$, $b = 58^\circ$, radius $= 25^\circ$, brightness range 2.5R



$b = 90^\circ$, radius $= 25^\circ$, brightness range 1.0R

What's next

After completing the discussed corrections to the program, we could use the cleaned integral map for re-calibration of spectra. This would make the data ready for applying PPV techniques like VCA.

A similar technique could be applied to recover another H_α survey, SHASSA, which also seems to have this problem: