ASTRO 735 - Problems

Set 3

Due: Mon, Oct. 23

Consider the following quantities which relate the metric and scale factor to observables such as apparent number, luminosity and size. We have defined the meaning and methods for finding these quantities as a function of z, Ω_0 and Ω_Λ in class.

		units
(i)	$dV = 4\pi D^2 dr$	$(c/H_0)^3$
(ii)	$V(z) = \int_0^z dV$	$(c/H_0)^3$
(iii)	D_L	(c/H_0)
(iv)	D_A	(c/H_0)

1. Determine (i)-(iv) in the appropriate units for the case when $\Omega_\Lambda = 0$ for $z = 1$ and 1000, for $\Omega_0 = 0.1, 0.3$ and 1. Specify the analytic formulae used. (I suggest you write general programs that allow you to do these computations for any z and any Ω_0; see problems 3 and 4.)

2. As above for $\Omega_\Lambda \neq 0$: $\Omega_0 + \Omega_\Lambda = 1$ (only two new cases). Please do this calculation by generalizing the programs (subroutines) you write for the above problem such that you can do your calculation for any Ω_Λ and any Ω_0. Specify the formulae you are using (in terms of integrals if the solutions are not generally analytic). Check any integration against cases that can be done analytically.

Summarize your results in tabular form for the above two exercises, using appropriate scientific notation and no more than 3 significant digits. Check your results; you aren't done until you get the right answer.

Comment on the differences between a given quantity at $z = 1$ for the different world models (i.e. different Ω_Λ and Ω_0). Do the differences depend on the total curvature ($\Re$), Ω_0, or both? What about at $z = 1000$? (Note that you have calculated 5 cases. Three cases have the same curvature, $\Re$; an overlapping set of three cases have varying curvature with fixed $\Omega_\Lambda = 0$; finally there are two pairs of cases with the same Ω_0 but varying $\Re$. This should be enough, but feel free to calculate other cases to check your conclusions.)

3. Cosmological leverage. Here you will be determining what the optimum redshift is for measuring Ω_Λ, in the absence of measurement error considerations. In class we demonstrated that at low z ($\ll 1$), the distance measure, D, depends linearly on z, i.e. is independent of the curvature and the deceleration (i.e. the matter density and vacuum energy). At very high redshift ($\gg 1$), we also argued that D depends only on the matter density today (Ω_0). (Longair shows that D = 2 c/H_0 Ω_0 at large z.) At intermediate redshifts, then, there must be leverage to determine Ω_Λ. The specific redshift that can differentiate best between world models depends on the range of values of Ω_0 *and* Ω_Λ being considered. Since at intermediate redshift q_0 dominates the behavior of D, the effects of Ω_Λ and Ω_0 will be coupled. If we do not have independent constraints on Ω_0 (i.e. from very high redshift), then we need to determine both Ω_Λ and Ω_0. We will assume here that we do not have any additional information about Ω_0. This means that more than one redshift (i.e. a range of intermediate redshifts) may be needed to distinguish between a wide range of parameters. These are generic statements. Here we will consider only the 5 specific cases from (1) and (2) above.

Form the ratio of the difference of each quantity (i)-(iv) between two cosmologies (one of which well represent a "fiducial" cosmology), to the quantity for the fiducial cosmology. Plot these ratios versus redshift. We will adopt $\Omega_\Lambda = 0.7$ and $\Omega_0 = 0.3$ as fiducial. Since luminosity is proportional to D_L, consider D_L^2 instead of D_L. To be specific, for the quantity of D_L = $D_L(\Omega_0,\Omega_\Lambda,z)$, calculate and plot $[D_L(x,y,z)^2 - D_L(0.3,0.7,z)^2]/D_L(0.3,0.7,z)^2$ for each of the other 4 cases of x = Ω_0 and y = Ω_Λ. For the other three quantities $Q(x,y,z) = D_A(x,y,z)$, $dV(x,y,z)$, or $V(z)(x,y,z)$, calculate $[Q(x,y,z)-Q(0.3,0.7,z)]/Q(0.3,0.7,z)$. For each quantity you will have 4 curves.

At what redshifts do you get maximum deviations (in an absolute sense) from unity for these ratios? Which quantity has the most leverage (the largest absolute deviations)? Could you have predicted this from the formulae used in (1) or (2)?

If you wanted to discriminate between the entire family of models considered here, what redshift range is ideal? What if you just want to determine the presence of absence of Ω_Λ for a given Ω_0? Choose different cosmologies from our set of 5 as "fiducial" if you think this will be helpful in presenting your argument.

4. Cosmological signal-to-noise. Think of the "leverage" you calculated in (3) as the "signal" distinguishing between different cosmological parameters. Now consider a model for the noise. The most intuitive case is for luminosity, and hence D_L^2, as follows. For a fixed observing time on a given telescope and instrument, the uncertainty ("noise") in a measurement of the apparent source flux, s, will depend inversely on this flux (smaller s gives larger uncertainty) if the source is unresolved. If the source is resolved, the uncertainty will scale inversely with s/θ, where θ is the apparent size of the physical scale, d, which characterizes the source.[1] (For this exercise d is arbitrary.) This means that if you modulate (multiply) the ratios you have calculated in (3) by s (unresolved source) or s/θ (resolved source), you will have a function that scales with redshift in proportion to the "signal-to-noise" ratio of a measurement attempting to distinguish between world models. We will simply refer to this as the cosmological "signal-to-noise" ratio (SNR).

Consider an observing procedures where the apparent source flux, s, will be observed in a fixed, observed band-pass, i.e. $s = s_{\Delta\lambda_0}$, as defined in class. You may find it useful to normalize your noise function ($1/s_{\Delta\lambda_0}$ or $\theta/s_{\Delta\lambda_0}$) at some fiducial redshift, say $z = 1$. Consider that we will be observing "standard" sources which do not evolve in physical size or luminosity. Finally, we specify that the fiducial cosmology is used to calculate the noise function.

(a) Calculate this cosmological SNR for the case of D_L^2, unresolved sources and the two world-models: ($\Omega_\Lambda = 0.0$, $\Omega_0 = 0.3$) *versus* ($\Omega_\Lambda = 0.7$, $\Omega_0 = 0.3$). Consider that sources' spectral energy distributions are characterized by $L_\nu \propto \nu^{-1}$ for the redshift range of interest.

How is the optimum redshift modulated by noise?

(b) Repeat the experiment for resolved sources.

(c) Repeat the experiment for resolved sources and $L_\nu \propto \nu^{-2}$ (typical of blue galaxies at optical to UV wavelengths) and $L_\nu \propto \nu^{-5}$ (typical of red galaxies at optical to UV wavelengths).

Conclude what is the optimum redshift for making cosmological measurements, whether you should be counting numbers in redshift shells, measuring apparent fluxes or sizes, examining red or blue sources. Which choices are the most important?

[1]The assumption here is that the source is in the back-ground-limited regime, i.e. its effective surface-brightness is less than that of the sky.