ASTRO 735 - Solutions

Set 2

(a) Recall

$$K(z) \equiv -2.5 \log \left[\ (1+z)^{-1} \ \frac{\int_0^\infty L_\lambda(\frac{\lambda}{1+z}, z=0) \ G(\lambda) \ d\lambda}{\int_0^\infty L_\lambda(\lambda, z=0) \ G(\lambda) \ d\lambda} \ \right].$$

If $L_\nu \propto \nu^{-\alpha}$ then $L_\lambda \propto \lambda^{2-\alpha}$, hence

$$\begin{aligned} K(z) &= -2.5 \log \left[\ (1+z)^{-1+2-\alpha} \ \frac{\int_0^\infty \lambda^{2-\alpha} \ G(\lambda) \ d\lambda}{\int_0^\infty \lambda^{2-\alpha} \ G(\lambda) \ d\lambda} \ \right] \\ &= -2.5 \ (1-\alpha) \log \ (1+z). \end{aligned}$$

The conditions $K(z) = 0$ is met for $\alpha = 1$, e.g., AGN/QSOs for observed bands and source redshifts such that $\lambda_e > 1216$ Å.

(b) Start with $I_{bol} = s_{bol}/\Delta\theta^2$ such that I_{bol} has units of ergs/s-cm^2-steradian. Using our definitions of s_{bol}, $\Delta\theta$ and d from class or Longair ($\Delta\theta$ and d are the apparent and physical source sizes, respectively):

$$I_{bol}(z) = \frac{L_{bol}}{4\pi D_L^2} \ \frac{D_A^2}{d^2} = \frac{L_{bol}}{4\pi d^2} \ (1+z)^{-4}.$$

Note the first term on the r.h.s. is a constant for a given, non-evolving source. Hence the redshift dependence of I_{bol} is only on kinematic factors of $(1+z)$ and not on the details of the metric (D).

For surface brightness measured in other units (I_ν, I_λ, I_γ), the only term that changes is L (L_ν, L_λ, L_γ). Using the same arguments in class for the different kinematic dependence of $s_\nu, s_\lambda, s_\gamma$:

$$\begin{aligned} I_{bol} &\propto (1+z)^{-4} \\ I_\nu &\propto (1+z)^{-3} \\ I_\lambda &\propto (1+z)^{-5} \\ I_\gamma &\propto (1+z)^{-3} \end{aligned}$$

(c) From our class notes (dropping subscript "o" which stands for "observed"):

$$s_{\Delta\lambda}(z) = \frac{L_{\Delta\lambda}}{4\pi D_L^2} \ \times \ k(z) \ \times \ e(z)$$

where $k(0) = e(0) = 1$. As before:

$$I_{\Delta\lambda} = s_{\Delta\lambda}/\Delta\theta^2$$

hence

$$\frac{I_{\Delta\lambda}(z)}{I_{\Delta\lambda}(z=0)} = (1+z)^{-4} \ \times \ k(z) \times \ e(z)$$

In magnitudes

$$-2.5 \log \left(\frac{I_{\Delta\lambda}(z)}{I_{\Delta\lambda}(z{=}0)}\right) \ = \ \mu(z) - \mu(z{=}0)$$

$$= \ 10 \log (1+z) + K(z) + E(z)$$

Assuming $L_\nu \propto \nu^{-\alpha}$ from (a) and dropping the evolutionary correction yields:

$$\mu(z) \quad = \quad \mu(z{=}0) + 2.5\ (\alpha+3) \log (1+z)$$

You will get the same result for I_λ, I_γ, and I_ν, as long as $L_\nu \propto \nu^{-\alpha}$ (equivalent to $L_\lambda \propto \lambda^{\alpha-2}$). In this case the apparent surface-brightness is independent of redshift for $\alpha = -3$.

The Rayleigh-Jeans limit for black-body radiation is as close as one gets to this condition for common, astrophysical sources. In this limit ($hc/\lambda \ll kT$) $L_\lambda \propto \lambda^{-4}$, i.e. $\alpha = -2$.