Lab 1. Positional Astronomy

1 Introduction

This two-week introductory lab will help you understand the most basic aspect of astronomy: observing the sky and understanding when and where a given celestial object will be visible. To do this, you must first know how the Earth's orientation and motion is reflected in the apparent motions of celestial objects, and you must learn the jargon astronomers use for precise positional astronomy. To do this, you will have to consider the geometry of spheres; the paper-and-pencil aspects of this lab, along with some simple experiments will help you do this.

This will be the approach of all the labs in this course: we will first develop some basic tools relevant to the problem at hand and then apply them to a real situation to obtain a result. In so doing, we will address two important issues: *1)* how are scientific methods and tools developed, and *2)* why is a particular method useful?

Another recurring theme will be that of error analysis. It is vital to discuss the types and degree of errors that may be present in scientific work, for a reader otherwise has no way to judge how reliable the stated result may be. In this lab, therefore, you will analyze the errors present in your observations: the techniques used in this lab will apply repeatedly in later labs.

2 Background

2.1 Positions in the Sky

One the surface of the Earth, we measure the position of a place using longitude and latitude. Longitude is measured by lines that extend from the north pole to the south pole, and are measured from 180° E to 180° W of a zero-degree (0°) from an imaginary line that goes through Greenwich, England. Latitude is measured by parallel lines that mark the number of degrees north or south of equator. The north pole is $+90°$ and the south pole is $-90°$.

Astronomers set up a similar set of coordinates in the sky (see Fig. 1). The *celestial equator* is an imaginary projection of the Earth's equator on the celestial sphere and the *celestial poles* are the imaginary points where extensions of the Earth's axis into space would meet the celestial sphere. A line that extends over the sky from the celestial north pole, through your zenith, to the celestial south pole is your *meridian*. The meridian that passes through Greenwich is called the *prime meridian*.

The astronomers' coordinate that is similar to longitude is called *right ascension* (denoted by α) and the coordinate that is similar to latitude is called *declination* (δ). Like longitude, right ascension is referenced to a fairly arbitrary line in the heavens: the meridian that was over Greenwich, England at noon, on March 21, 1900. (This is sometimes called the *first point of Aries.*) Unlike longitude, however, astronomers choose to measure α in *hours* (as opposed to degrees of longitude), with 24

Figure 1: Celestial Sphere

hours equalling 360 degrees. Practically speaking, this means that at noon on the vernal equinox, $\alpha = 0^h$ is approximately on your meridian, and each hour thereafter α increases by about an hour. Declination, on the other hand, is exactly analogous to latitude, and is measured in degrees from the celestial equator, from $-90°$ at the south pole to $+90°$ at the north pole.

Right ascension and declination are fixed to the heavens, hence every object has a unique position, (α, δ). However, in order to describe the apparent position of an object on the sky *at a given time* we must use the horizon coordinate system (Fig. 2). *Altitude* is one horizon coordinate, and is the vertical angle between the horizon and the object on the sky. Objects directly over your head (on the *zenith*) have an altitude of 90°. The other coordinate is *azimuth* which is the angular distance measured eastwards along the horizon from north to the vertical circle running through the object (*i.e.,* the arc describing the altitude). Objects that are due east have an azimuth of 90°; when an object is due west, it has an azimuth of 270°. Obviously, altitude and azimuth depend on your position on earth, while the *equatorial coordinates,* right ascension and declination do not.

Another term that is very commonly used is *hour angle.* It is measured in units of time westward from your meridian, and is the amount of time that has elapsed (or will elapse) from the object last *transited,* or crossed your local meridian. For example, an object that is just rising in the east may have an hour angle that is about 6 hours, east.

Figure 2: Horizon Coordinate System

2.2 Time

The rotation of the Earth on its axis and its revolution around the Sun cause the positions of the stars to change constantly. It is obviously necessary to have a precise definition of the current time, and Question 1 illustrates why.

Question 1 *Some telescopes can be moved by computer control to a given location in the sky, say a star an astronomer wishes to observe. Such telescopes sometimes have acquisition TV monitors with a field of view of about 50 arc seconds (1 arc second = 1/60 arc minute, which in turn equals 1/60 of one degree of arc). Now, on the celestial equator, how many arc seconds of right ascension does the rotation of the Earth cause to pass by a given point in one second of time? If the astronomer's clock is off by 10 seconds, should he expect to see the star on his TV when the telescope moves there? Do you expect the same result if the astronomer's object is near the north celestial pole?*

Astronomers use a number of different timescales; we will consider three in this lab.

Local Standard time (LST) is the obvious one: this is the current time of day at your longitude.

Universal time (UT) is often more useful than local time. It is defined as the current local time at the prime meridian, *i.e.*, the local time at Greenwich, England. A rule of thumb for the

State College area is that UT is always five hours ahead of local time (five being the number of timezones between here and Greenwich). UT is important because there is only *one* UT anywhere in the world, whereas every timezone has its own local time.

Sidereal time (ST) is defined as *the hour angle of vernal equinox.* Thus, sidereal time is fundamentally different from local and universal time: while the latter two are referenced to the Sun (*i.e.,* the Sun is always highest in the sky around noon), sidereal time is referenced to the celestial sphere, and thereby to the stars. (Thus the name sidereal, which means “stars”.) At noon on the vernal equinox, the sidereal time is approximately 0^h. Thereafter, the sidereal time changes by about $24^h3^m56^s$ for every 24^h solar time. (Why?)

Question 2 *Use Fig. 1 and the definitions of right ascension and hour angle to show that ST also equals the right ascension currently on your local meridian.*

3 Equipment

3.1 A plumb line

A plumb line is a line that hangs precisely vertically; it is easily constructed by attaching a moderate weight to the end of a string. If you hold it or affix it so that it is motionless with the weight suspended above the ground, it will hang straight up and down. Make one of these.

3.2 A cross-staff

Cut a piece of fairly sturdy cardboard into the shape shown in Fig. 3. The dark area in the middle represents a hole that fits snugly over a meter stick. The cardboard piece can then be slid up and down the meter stick, and as you will see, it makes a simple tool for measuring angles in the sky.

4 Lab

Our goal of this lab is to attach some real data to the abstract figures given above. Proceed through them in order; note that this lab requires a clear night and you should therefore plan ahead and be ready to take advantage of clear skies. This is particularly important in central Pennsylvania: in the winter the probability of getting a clear night is small and the probability of getting a clear night on Tuesday is small divided by seven. **Be prepared.**

4.1 Measuring sidereal time

The first step is to go out on a clear night and determine the sidereal time. From the definition of sidereal time given above, we know that it is equal to the right ascension currently at the local

Figure 3: Shape of the cardboard slider on the cross-staff.

meridian. An obvious way to determine this is to observe a star and note the LST at which it transits. You then can just look up the right ascension of the star, and you know the sidereal time at the LST on the particular date you observed.

The local meridian clearly runs from due north through the zenith to due south; you therefore need some way to be sure you are sighting along a true north-south line. This is where the plumb line comes in. Mark a line on the ground that runs perfectly north-south. (Use Polaris, the pole star, to make sure of your directions. One way to make the line is with a piece of string tied to two stakes which can be driven into the ground.) Then hold your plumb line in front of you so that part of it is projected against the sky and part against the line on the ground. Sight along it so that it lines up with your north-south ground line; the part projected against the sky will track the local meridian. When a star passes behind your plumb line, it has transited.

Question 3 *On what date did you make the observation? At what local time did your star transit?*

Question 4 *What was the Universal time at the transit?*

Question 5 *What was the sidereal time at the transit? How accurate do you believe your measurement to be?*

Question 5 raises a small detail that we have overlooked so far: you have to know what star you observed. Figure out what it was. As long as you used a relatively bright star, you should have little trouble tracking down in an atlas what you were looking at. Then you can look up the right ascension of the star in the *Astronomical Almanac* or some other reference book, and by definition

Figure 4: Using the cross-staff.

you have the precise sidereal time, ST, at your LST on the date of observation.

Question 6 *What star did you observe? What constellation is it in?*

4.2 Angular distances in the sky

Your cross-staff is a device for measuring angles (either on earth or on the sky). Use it by holding one end of the meter stick up to your eye and moving the cardboard slider until the corners (either the two inner edges or the two outer edges) line up with the edges of your targets (see Fig. 4). If d is *half* the corner separation (either 3 cm or 1 cm) and m is the distance along the meter stick from your eye to the slider, then the angular extent or separation between two objects is

$$\tan \frac{1}{2}\theta = d/m \tag{1}$$

which, for small angles, reduces to

$$\theta \approx 2d/m \tag{2}$$

Go outside on a clear night and use the cross-staff to measure some angles in the sky. If the Moon is up, measure its angular extent. (Watch out! Depending on the phase of the Moon, you may have to do it during the day.) Also measure the angular separation of the stars in either the Big Dipper or Cassiopeia (do whichever is easier). Diagrams of these constellations[1] are given in

[1]The Big Dipper by itself is not a complete constellation; it forms part of *Ursa Major,* the Big Bear. Nevertheless, it is one of the most familiar figures in the sky.

Fig. 5; you need only do whichever one is easier to see. Make measurements by moving the slider until the Moon or the star pair lines up with two corners, and note down the value of m in a table.

Measure each separation several times. For each pair of stars and the Moon, make your first measurement, move the slide out of place, move it back and repeat the measurement. Write down each value in a table, making sure to note which measurement corresponded to which star pair. The values of m you get will be close to one another, but they will probably not be identical. When you are done, you should have several tables of m, one for each star pair (or the Moon). Be sure to note which object(s) are being measured and whether you were using the inner or outer corners of your cross-staff. For each series of measurements, calculate the average value and record it. The average angular extent $< \theta >$ is given by

$$< \theta >= \frac{\sum_{i=1}^{N} \theta_i}{N} \tag{3}$$

where N is the total number of measurements and the θ_i are the individual values derived from your measurements.

Question 7 *On what date did you make the observation? Which two stars did you observe? At what local time and Universal time did the first star in the pair transit? (If you didn't observe transit, calculcate it from what you have already determined in 4.1.)*

Question 8 *What are the coordinates (α, δ) of that star?*

Question 9 What was the sidereal time at the transit? How accurate do you believe your measurement to be?

Question 10 *What are your various average values?*

4.3 Analysis and quantification of errors

Unfortunately, your answers to the above question are almost worthless as stated. The problem is that you have no idea how reliable the stated value is: it might be quite accurate, or it might not. Without some quantification of the errors present in the measurements, there is no way to know this.

A fundamental quantity for describing the uncertainty in the data is the *standard deviation,* denoted by σ. This may be calculated using the following equation:

$$\sigma^2 = \frac{\sum_{i=1}^{N} (\theta_i - < \theta >)^2}{N-1} \tag{4}$$

where the symbols have the same meaning as in the equation for the average. The value of σ describes the uncertainty of *one* of your measurements; all things being equal, 68% of your mea-

Figure 5: The Big Dipper (top) and Cassiopeia (bottom).

Figure 6: An additional source of error in the cross-staff measurements.

surements should lie within 1σ of the average, and 99.7% should lie within $3\,\sigma$. The uncertainty of your *average* value is then

$$\sigma_{\text{average}} = \sigma/\sqrt{N} \tag{5}$$

A large σ therefore indicates a wide distribution of measurements and a poor determination of the average.

Question 11 *Calculate σ for your measurements of the Moon and for one pair of stars in the constellation you observed.*

Question 12 *The distance to the Moon is 3.8×10^5 km. What is your mean and error in km, for the size of the Moon?*

There is a further source of error to consider as well. We have quantified our *measurement* errors; that is, random errors introduced by variations in your positioning of the slider. We now must quantify *instrumental* errors, which are systematic errors introduced by the cross-staff itself. The most likely such error is produced by the cardboard slider, which does not alway remain perfectly perpendicular to the meter stick. A tilt like the one shown in Fig. 6 will add an additional offset to your measurement.

How can we deal with this? First, we must analyze the degree of the error. Set your cross-staff between two tables so that the middle portion, including the slider, is suspended with nothing underneath it. Then make 15 of the following measurements: move the slider 20 cm or so, and use your plumb line to measure the offset between the upper edge and the edge next to the meter stick.

Record the offset value and repeat.

Question 13 *What is the average offset (call it Δ) and the standard deviation σ_Δ of the offset values?*

if $\Delta \neq 0$, then there is an additive offset in your values of m.

Question 14 *Revise your average values if necessary to account for the nonzero value of Δ*

But what is the new uncertainty? You did not just add Δ: you also factored in an additional uncertainty, σ_Δ. Clearly your original σ will be altered.

Here is the situation: we now have two parameters instead of one (m and Δ rather than just m). For this experiment we will assume that the fluctuations in each parameter are completely *uncorrelated.* In other words, errors in the measurement m are unrelated to the tilt of the slider. In this case, the overall standard deviation σ_{tot} is produced by adding the individual deviations in quadrature:

$$\sigma_{tot}^2 = \sigma^2 + \sigma_\Delta^2 \tag{6}$$

Question 15 *What is σ_{tot} for your observations of the Moon and the one star pair? What is your new uncertainty in km for the diameter for the Moon?*

This phenomenon is called *propagation of errors.* You will encounter it frequently, since many physical situations have several parameters, each with its own uncertainty.

5 Concluding remarks

This exercise has served two purposes, both central to astronomy.

It is obviously vital to have a working knowledge of how the sky's appearance changes during the night. We have put together the basic geometry in the first part of this lab. Although there are many details we did not touch on, they derive from the essentials given here.

It is also vital to be aware of any errors that may have crept into your work. These take the obvious form of errors in measurement, as well as the slightly more subtle form of minor equipment deficiencies. As you proceed in this course, you should be constantly watching for possible errors and if you cannot remove them, you should at least assess carefully their impact on your results.